Image Re-Sampling

Reconstruction
Sampling
Re-Sampling

1. Re-Construction
   Which one?
   
   ① device-oriented (splat) - overlap
   ② connect the dots
      piecewise constant
      linear
      splines - overlap / support
   ③ as smooth as possible

Math:

Samples = Spike Chain \rightarrow \text{Convolution}

\begin{array}{c}
\text{Sliding window of kernel: } \ast \\
\text{How to compute @ pos } X
\end{array}

Why this way?
What kernel?
Sampling Theory

How to make reconstruction unambiguous?

Consider only sine waves

\[
\begin{align*}
0 & \quad 2\pi & \quad 1 & \quad 2\pi & \quad 2 & \quad 2\pi + \n\end{align*}
\]

Any will pass through these points

- need to restrict to < \(1\)

Intuition: "smooth" signals can't turn around too often between samples

(less than \(1\) min/max per period)

Note: if you have more than 2 samples per period

- lowest frequency is unique

- aliases with multiples of freq

Fourier Theory

Represent any signal as sum of sine waves

- Not quite, some complications

\[
f(t) = \sin 1 \ 2\pi t + \frac{1}{2} \sin 3 \ 2\pi t + \frac{1}{3} \sin 6 \ 2\pi t
\]

\[
F(w) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T
\]

High frequencies \(\approx\) abrupt changes

- need to use high frequencies to make sharp changes
Sampling Theorem (Nyquist Shannon)

If Signal is Band Limited ($\omega_{HF} < \omega$) you know the highest frequency in it
AND you sample $\Omega > 2\omega$

THEN $\omega$ you can exactly reconstruct

Ideal Reconstruction
determine the signal that fits samples (that doesn't exceed the band limit)

make spike chain (lots of HF)
low-pass filter (remove HF)

Filtering is convolution