1. MULTIPLE CHOICE SECTION. Write the letter of the correct answer (or answers) in the space provided. You do not need to give a justification for your answer(s).

(1) Consider the ellipse given in parametric form as \( C(t) = (2 \cos t, \sin t) \), for parameter values \( 0 \leq t \leq 2\pi \). Which of the following vectors is normal to this curve at the location corresponding to \( t = \pi/4 \)?

[Friendly reminder: \( \sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2 \)]

Write the letter of the ONE most correct answer here: 

(a) The vector (2,1).
(b) The vector (−2,1).
(c) The vector (1,2).

(2) When multiplying transforms together, the order of multiplication generally does matter. There are exceptions, however. In which of the following scenarios would we get the exact same result regardless of the order in which we do the multiplication?

Write the letters of ALL correct answers here: 

(a) When multiplying 3D rotations that are both around the same axis.
(b) When multiplying a translational transformation with an (axis aligned) scaling transformation.
(c) When multiplying two (axis aligned) scaling transformations.
(3) Does a curve that has been arc-length parameterized actually look different than one that has not?

(Circle or underline the ONE most correct answer)

(a) Yes. An arc-length parameterized curve is clearly smoother than one that has an arbitrary parameterization.
(b) Maybe. It could look different, if the curve is not C(1) continuous.
(c) No. A curve can be converted from an arbitrary parameterization to arc-length, and vice versa, with no change to the apparent shape of the curve itself. It would take a dynamic behavior (e.g., observing the motion of an object along the curve) to visually appreciate the difference between the 2 parameterizations.

(4) Which of the following statements about mipmaps are correct?

(Circle or underline ALL correct answers)

(a) Mipmaps help eliminate aliasing, while only requiring a minimal memory overhead to store a hierarchy of texture resolutions.
(b) A disadvantage of mipmaps is that discontinuities are very easily visible when a texture transitions between two different resolutions of the mipmap.
(c) We can get a very comparable effect to the use of mipmaps by rendering with a standard texture and selectively blurring the resulting image at locations where the texture resolution is significantly finer than the image pixel resolution.
2. Answer the following questions in no more than 1-3 sentences (when required) or by filling in the answer boxes.

(a) There are reasons both for and against either the forward or the backward ray tracing approach.

i. State one factor that weighs in favor of forward ray tracing.

ii. State one aspect that makes backward ray tracing more desirable.

(b) A parametric line in 3D is given by the expression \( C(t) = p_0 + t d \) where \( p_0 \) and \( n \) are 3D vectors. Show that if \( d \) has unit magnitude (i.e. it is a normalized vector), then \( C(t) \) is arc-length parameterized.

(c) In local illumination models (e.g. the Phong model we discussed in class) shading of a given point on a visible surface object is computed independently of how any other surface has been shaded. Describe two examples of scenes or phenomena that a local illumination model would not be able to realistically reproduce.
(d) If $C(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$ is a curve in 3D, show that the vector $\begin{pmatrix} y'(t) + z'(t) \\ -x'(t) \\ -y'(t) \end{pmatrix}$ is always perpendicular to it. [Hint: How does it relate to the tangent?]

(e) Consider the following scene. A light source is placed at location $(10, 15, 0)$ (illustrated as the star). The camera (illustrated as the eye) is placed at $(40, 15, 0)$. The ground plane (shaded grey) is at $Y = 0$. [For this question we are only considering points along the plane $Z = 0$.] Assume we are using a Phong lighting model, which incorporates ambient, specular and diffuse components.

i. On which location of the ground plane will the specular component have its brightest value? Mark this location directly with an “X” in the illustration above.

ii. On which location of the ground plane will the diffuse component have its maximum value? Mark this location directly with an ”O” in the illustration above.
3. Consider the four points $P_0(0,0)$, $P_1(8,0)$, $P_2(4,4)$, $P_3(8,4)$ as shown in the illustration below:

We consider the cubic Bézier curve defined by these four points.

(a) Compute the location of the points on the curve corresponding to parameter values $u = 0$, $u = 0.25$, and $u = 0.5$.

(b) Approximately sketch the cubic Bézier segment generated by these points (the values from the previous question should be of help). Draw your curve directly on the figure above.
4. In the recursive ray tracing algorithm, we allow light to travel from the light to the camera via a number of bounces. Remember that, with the exception of the bounce closest to the light source (which can incorporate both diffuse and specular reflection), all other bounces are purely specular (mirror reflections).

The illustration below illustrates a camera (on the left), a light source (drawn as a star), and an environment consisting of opaque walls (shaded grey).

(a) Draw a light path with exactly two bounces, that connects the camera to the light source (without shadowing). **Clearly mark any angles along the path that are assumed to be equal.**

(b) Draw a light path with exactly three bounces, that connects the camera to the light source (without shadowing). **Clearly mark any angles along the path that are assumed to be equal.**
5. Consider a *quadratic* parametric curve $C(u) = a_0 + a_1 u + a_2 u^2$ in 3 dimensions, which satisfies the following constraints:

- $C(0) = p_0$
- $C''(0) = 4(p_0 - 2p_1 + p_2)$
- $C(1) = p_2$

where $p_0, p_1, p_2$ are control points given as input.

Compute the basis matrix $B$ that would allow us to express this curve in the form

$$C(u) = \begin{pmatrix} 1 & u & u^2 \end{pmatrix} B \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p \end{pmatrix} = uBP$$