

Linear Algebra Refresher

Please either answer on additional sheets. Be sure to write your name AND CS login on EVERY page, and to attach multiple pages together with a staple.

This is due in class at 9:30 AM on Tuesday, September 13th.

The Bonus questions are hard - don't worry if you can't get them, but thinking about them will be good practice.

1. Consider the 3×3 matrix $\begin{matrix} & x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 & . \\ 1 & 1 & 1 & \end{matrix}$.
- What is the determinant of this matrix?
 - Show that if the three points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) are colinear that the determinant is zero.
 - BONUS: Show that if the determinant is zero, the three points must be colinear.
2. Suppose that we have n $k \times k$ matrices $(\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n)$.
- Consider multiplying these matrices by a k -vector - that is computing $\mathbf{A}_1 \mathbf{A}_2 \cdots \mathbf{A}_n \mathbf{x}$. Computing this product left to right $((\mathbf{A}_1 \mathbf{A}_2) \mathbf{A}_3) \cdots$ gives the same answer as computing it right to left $\cdots (\mathbf{A}_{n-1} (\mathbf{A}_n \mathbf{x}))$. However, the latter is probably faster, especially if we only count the cost of the arithmetic operations. How much faster will it be if we only count multiplies? (write a ratio in terms of k and n)
 - If we were to multiply each of a set of m vectors $(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m)$ by the product of all of the matrices, it might be faster to multiply all the matrices together first, then multiply this combined matrix by all of the vectors. Write an expression that decides which way is better (again, consider only the cost of multiplication). Your answer should be an "if" statement based on n , m , and k .
3. Consider the plane (in 3 space) $2x + 2y + z = 2$. What point on this plane is closest to the origin? Does the unit sphere intersect this plane?
4. For the matrix $\begin{matrix} 1 & 0 & 0 \\ r & s & s \\ x & y & z \end{matrix}$ to be orthonormal, what are the possible values of r, s, x, y , and z ? How many (real-valued) answers are there? Which ones are "right handed" and which ones are "left handed"?
5. Let \mathbf{p}_0 , \mathbf{p}_1 , and \mathbf{p}_2 be non-colinear points in 3-space. These points define a plane.
- Consider the cross product $\mathbf{n} = (\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)$. What is the geometric intuition for \mathbf{n} ?
 - For another point \mathbf{p}_3 also on the plane, what is $\mathbf{n} \cdot (\mathbf{p}_3 - \mathbf{p}_0)$?
 - Typically, the equation of a plane is written as $ax + by + cz + d = 0$, where $\mathbf{p} = (x, y, z)$. What are the values of a, b, c and d in terms of $\mathbf{n} = (n_x, n_y, n_z)$ and $\mathbf{p}_0 = (p_x, p_y, p_z)$? (Hint: Use the answer to part B above. \mathbf{n} and \mathbf{p}_0 are the same variables as in part A.)