

FOURIER INTUITIONS

Low / High Frequencies

Sharp Edges \Rightarrow HF

What it looks like in 2D

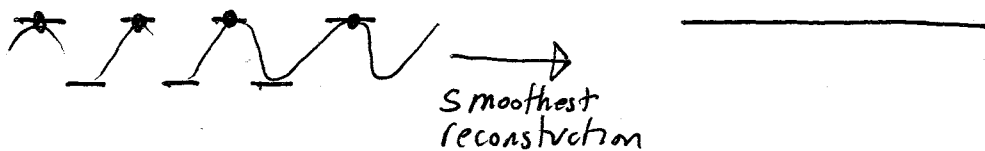
Band Limited

Sampling Theorem

If signal contains no frequencies higher than B
then it can be perfectly reconstructed from samples
at a rate of $2B$

if you have a sampling rate of S
then you can sample signals with band limits
 $< \frac{S}{2}$

if you have a signal with high frequencies
and you sample at a rate that is too
low, you get aliasing



Ideal reconstruction - signal that interpolates samples
but meets band limit

11/5

What if your signal is not band limited appropriately?

- raise sampling rate (can't)
- sample (and alias*)
- get rid of HF BEFORE sampling
throws away info - but sampling will too

PRE-FILTERING

Intuitively -

Signal is too sharp (has HF)

- get rid of sharpness (blur)
- spreads things around so you don't miss it

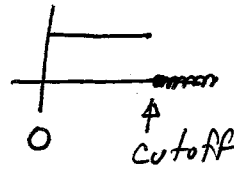
$$\begin{array}{cccccccc} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ & \uparrow & & \uparrow & & \uparrow & & & \\ 0 & .5 & 1 & .5 & 0 & & & & \end{array} \quad \begin{array}{l} \leftarrow \text{subsample by } 2 \\ \leftarrow \text{sub sample by } 3/2 \end{array}$$

Blur = convolve w/ "Blur kernel"
 $\frac{1}{2} [1 \ 2 \ 1]$

$$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \dots$$

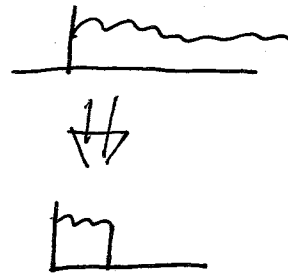
Low - Pass Filters

In frequency Domain



looks like a box

Multiply by spectrum

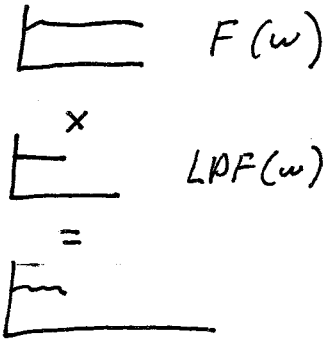


$$\frac{\text{IDEAL}}{\text{LPF}}$$

How do we do this

Time Domain
 $f(t)$

Fourier Transform
 \Rightarrow



filtered $f(t)$

\Leftarrow
 Fourier Transform

Instead :

Fourier Transform Multiply \Rightarrow Convolution

LPF \Rightarrow inverse fourier transform of LPF

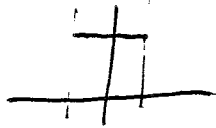


REAL VS IDEAL LPF

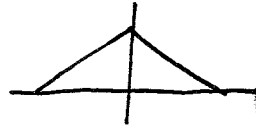
Ideal LPF isn't so ideal....
infinite extent
rings (overshoots)

Approximate

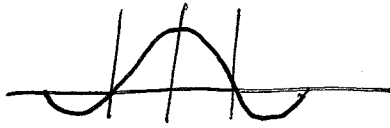
box



tent



cubic spline



Lancos2 - truncated sinc

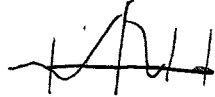
1/5

IDEAL RECONSTRUCTION (and Real)

samples are spike chain 

very sharp (spike chain \Leftrightarrow spike chain)

smooth it! Ideal LPF \rightarrow ideal reconstruction



more likely - use a "real" LPF that approx

FOURIER DETAILS