

CS559 Midterm Exam

October 20, 2009, 7:15pm-9:15 pm

This exam is closed book and closed notes.

You will have the entire period (until 9:15) to complete the exam, however it was designed to take less time.

Please write your name or CS login on **every page!** (we may unstaple the exams for grading). Please do this first (and check to make sure you have all of the pages)

Write numerical answers in fractional form or use radicals (square root symbols) – we would prefer to

see $\frac{\sqrt{3}}{2}$ than .866. You should not need a calculator for this exam.

Unless otherwise noted, assume that everything is a right-handed coordinate system and that angles are measured counter clockwise. E.g. to find the direction of rotation, point your thumb along the axis and curl your fingers.

If you need extra space, use the back of a page, but clearly mark what everything is. We may look at your work to determine partial credit. Not providing an answer is not the same as giving a wrong answer. For “forced choice” questions (multiple choice, true false, etc.), wrong answers will be penalized more than no answers.

If you feel a question is ambiguous or requires more information for you to make a good answer, you can state any additional assumptions you needed to arrive at your answer. However, you should probably avoid this: usually it is more a sign of a lack of thought about the question or understanding of the material.

Total : 100 pts.

Bonus for following directions: ___ / 3 pts

Question 1: ___ / 10 pts

Question 2: ___ / 7 pts

Question 3: ___ / 12 pts

Question 4: ___ / 10 pts

Question 5: ___ / 6 pts

Question 6: ___ / 15 pts

Question 7: ___ / 7 pts

Question 8: ___ / 12 pts

Question 9: ___ / 6 pts

Question 10: ___ / 12 pts

Question 1 (10 pts)

Consider a Quadratic Curve segment. It is parameterized by its starting point ($u=0$), the derivative at the starting point ($u=0$), and its endpoint ($u=1$).

The Basis Matrix for this curve segment (we called this B in class) is:

The Constraint Matrix for this curve segment (we called this C in class) is:

Hint: The Constraint Matrix isn't useful

Consider one of these curves in 2D with controls (P): (1,1), (1,-1), (3,1)

Hint: To make sure you understand this, make sure that you can compute the position of the curve at ($u=1$) since you know what the right answer should be.

Check UBP $u=[1 \ 1 \ 1] = (1-1=0) P_0 + (1-1) P_1 + 1 P_2 = P_2$ (curve does interpolate P_2)

A: What point does the curve pass through at $u=.5$?

$$\text{UBP } (u=[1 \ \frac{1}{2} \ \frac{1}{4}]) = (1-\frac{1}{4}=3/4) P_0 + (\frac{1}{2}-\frac{1}{4}=\frac{1}{4}) P_1 + \frac{1}{4} P_2 = \frac{3}{4} + \frac{1}{4} + 3 \cdot \frac{1}{4}, \frac{3}{4} - \frac{1}{4} + \frac{1}{4} = (1 \ \frac{3}{4}, \ 3/4)$$

B: What is the tangent vector to this curve at $u=1$?

(UBP)' = U'BP (only U depends on u, so its OK just to take the derivative of that)

$$U' = [0 \ 1 \ 2u], [0 \ 1 \ 2] (@u=1)$$

$$U'BP = [0 \ 1 \ 2] B P = -2 P_1 - 1 P_2 + 2 P_3 = -2-1+6, -2+1+2 = (3, 1)$$

A very different, but cool way to get at it:

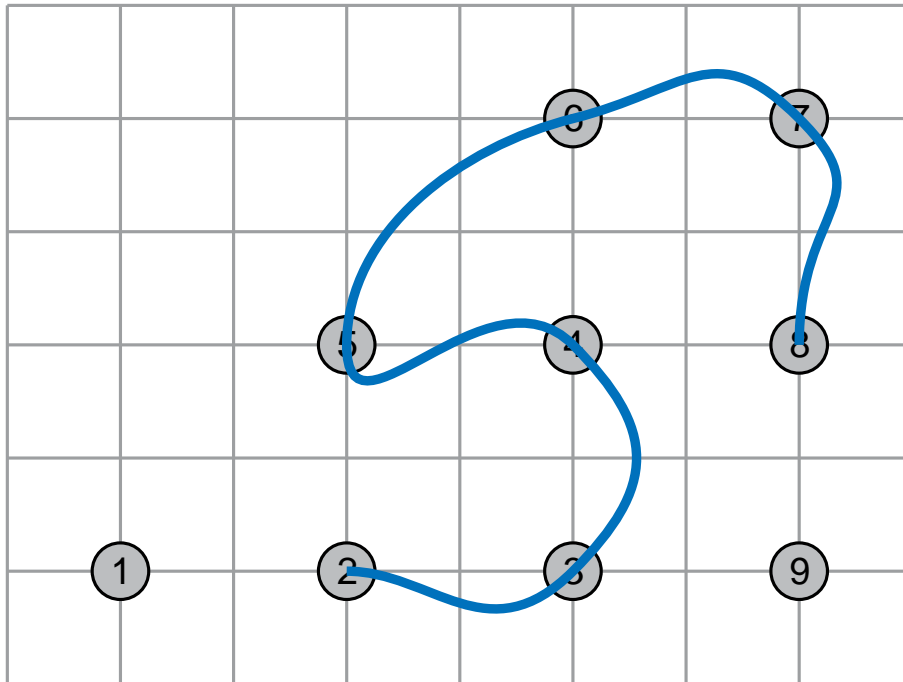
$f'(0) = P_1$, so the Bezier control points are $P_0, P_0+1/2 P_1, P_2$, or (1,1), (3/2,1/2), (3,1)

use DeCasteljau to compute the ($u=.5$) point ($P_{01} = 1 \ \frac{1}{4}, 3/4, P_{12}=2 \ \frac{1}{4}, 3/4, P_{123} = 1 \ \frac{3}{4}, \frac{3}{4}$)

$$f(1) = 2 (P_2 - P_1) = 2 (3/2, \frac{1}{2}) = (3, 1)$$

Question 2 (7 pts)

Sketch a Catmull-Rom (e.g. Cardinal Cubic with tension 0) spline through the following sequence of points (the points are in order as numbered 1-9):



Begin at 2

End at 8

Interpolate 2-8

Derivatives correct (dip 2-3, overshoot 3-4, zigzag at 4-5, straight into 8)

Question 3 (12 pts)

A: A Bezier Quadratic curve segment has its control points at $(0,0)$, $(0,1)$, $(2,1)$. Give the control points for a Bezier Cubic curve segment that covers the same curve.

Remember, quadratic $F'(0)=2(P2-P1)$, for cubic $F'(0)=3(P2-P1)$

$(0,0)$ $(0,2/3)$ $(2-4/3 = 2/3,1)$ $(2,1)$

2pts for the endpoints, 2 points for each of the derivative points

B: What is the position of this cubic curve at $(u=.5)$?

Do DeCasteljau on the Quadratic (because it's the same curve as the cubic, and easier to compute)

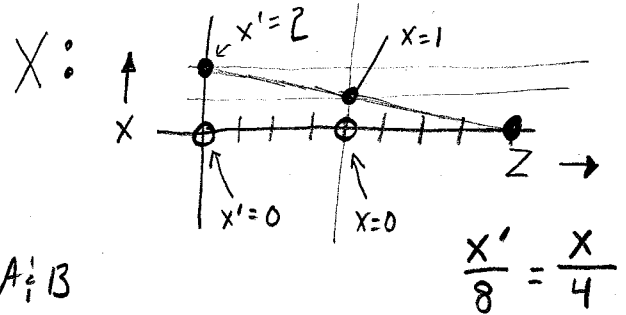
$P12 = (0,1/2)$ $P23=(1,1)$

$P123 = (1/2, \frac{3}{4})$

Question 4 (10 points)

Given that Halloween is coming (and is a big deal in Madison), we have decided to make shadow puppets!

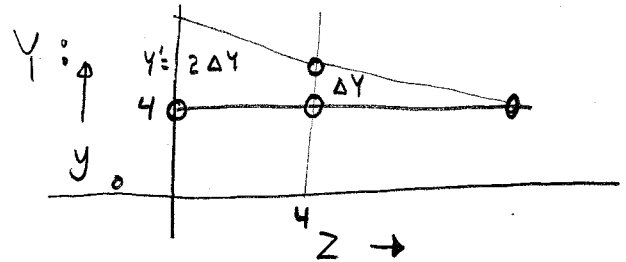
- A lightbulb is placed at 0,4,8.
- The first finger extends from 0,4,4 to 0,5,4.
- The second finger extends from 0,4,4 to 1,5,4.
- Everything is projected onto the wall z=0.



See pictures to get $x'_i y$ for $A_i B$

A) where does the first finger project to? (where do the endpoints go)
(the answer should be the positions of 2 points)

$(0, 4, 0)$ $(0, 6, 0)$

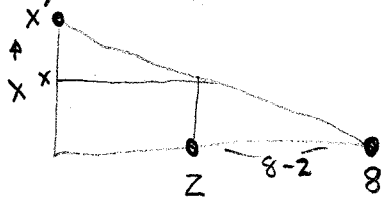


B) where does the second finger project to?
(the answer should be the positions of 2 points)

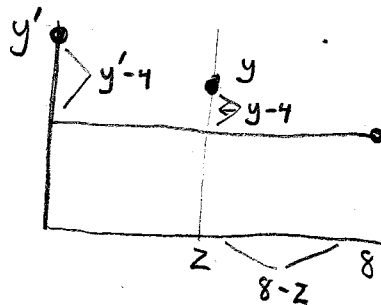
$(0, 4, 0)$ $(2, 6, 0)$

1 point for each of the 3 different points
(since 0,4,0 appears in both answers)

C) write a 4x4 projection matrix that determines where a point $(x,y,z,1)$ will be projected to.
(the answer should be a 4x4 matrix)



$$\frac{x'}{8} = \frac{x}{8-z} \quad x' = \frac{8x}{8-z}$$



$$\frac{y'-4}{8} = \frac{y-4}{8-z} \quad y' = \frac{8y-32}{8-z} + 4$$

$$y' = \frac{8y - 4z - 32 + 32}{8-z}$$

$$\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 8 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 8 \end{bmatrix}$$

$$z' = 0$$

2 points for x,y,w rows
1 point for z row
partial credit (half) for just having the similarity equations

Question 5 (6 pts)

If we use Brezenham's line drawing algorithm to draw the lines between the points below, how many pixels will get "set". (Note: Brezenham's algorithm is also called the Midpoint algorithm).

Hint: you should count the beginning and end points.

There is one pixel per row/column in the major direction.

a) from (10,10) to (20,10)

11

b) from (10,10) to (20,15)

11

c) from (10,10), to (20,20)

11

d) from (10,10) to (20,25)

16

3 points for have $a=b=c$
1 point for d being different than a,b,c
1 point for having 11 as the answer for a-c
1 point for having 16 as the answer for d

Question 6 (15 pts)

Imagine a simple OpenGL-like graphics toolkit. All of the transformation operations are like OpenGL in that there is a matrix stack and the command affect the top of it. Rotation is measured in degrees counterclockwise. For each of the little programs below, assume that the programs start out with the identity matrix on the stack. (from 2006 answer key)

Here is an example program and its resulting picture:

<pre> Draw Square PushMatrix Scale(2,2) Translate(2,1) Scale(-1,1) Draw Square PopMatrix Rotate 90 Translate -3,0 Draw Square </pre>	
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A: If we forgot the “Push Matrix” and “PopMatrix” commands in the example program, the last square would appear in a different place. Draw it on the example above.

B: If we had forgotten the “PushMatrix” and “Pop Matrix” commands (as in part A), and wanted to “reset” the transformation back to the identity at the end, we could add the following 3 lines of code. (in other words, if we added a DrawSquare command, it would place the square where the first line of the program does

Specify what the values of A, SX, SY, TX, TY should be:

Rotate A	A = -90
Scale SX, SY	SX = -1/2, SY = 1/2
Translate TX, TY	TX = -4, TY=4

Question 6 (continued)

C: Write a program that draws the following picture (with 3 squares) without using PushMatrix or PopMatrix commands. You can use any of the other commands from the example program.

	<p>Note: there are many possible answers, this is just one:</p> <pre> translate(5,0); draw Square(); translate(-2,0); scale(2,2); draw Square(); scale(1.5,1.5); translate(-1,0); draw Square(); </pre>
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Question 7 (7 pts)

- A) A 2D linear transformation is a rotation that transforms the point (5,0) to (3, 4). Give the 2x2 matrix for this rotation.

The x-axis goes to 3,4 (normalized), the Y axis must be perpendicular and properly handed.

$3/5$	$-4/5$
$4/5$	$3/5$

4 pts (1 point for each matrix entry)
3 pts for knowing rotation is unique

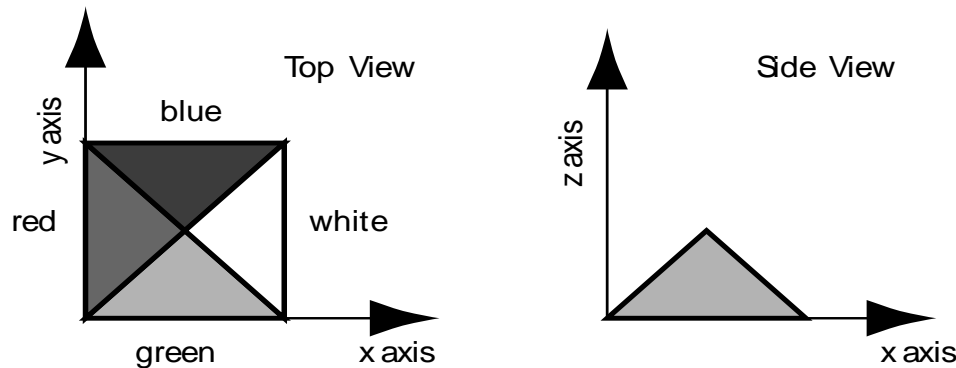
- B) How many right answers are there to part A?

Exactly 1 (if you rotate around the circle many times, you still end up with the same transformation)

Question 8 (12pts)

Assume that the x-axis (for the world) points east, the y-axis points north, and the z-axis points up. (this is a right-handed coordinate system)

Imagine a pyramid with a square base placed with its corner at the origin. Each of the triangular sides of the pyramid is painted a different color. The length of a side of the pyramid is 2 units, so the top of the pyramid is at (1,1,1).



Yes, we know Part A was hard
That's why we grade on a curve.
We also will be generous with partially correct answers.

Assume that the film plane or screen is the “normal” xy plane with x going to the right and y going up. The center of the film plane is at the origin, and the viewer sights down the negative Z axis.

5A: Give an orthographic projection matrix (a 4x4 matrix) that gives a view of the white side of the pyramid. Don't worry about depth in the viewing volume other than that objects that are farther away

	<p>The first column says where the x axis goes... The negative X axis should go into the screen (-Z) so pos X goes to pos Z The Y axis should go to the right (X) The Z axis should go up (Y) We should be sitting on the positive X axis, so the origin (4th column) should go “deep” into the screen (negative Z)</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td><td>-2</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>1</td></tr> </table> <p>(any negative value for the -2 is OK, actually any value is OK if you don't have occlusions)</p>	0	1	0	0	0	0	1	0	1	0	0	-2	0	0	0	1
0	1	0	0															
0	0	1	0															
1	0	0	-2															
0	0	0	1															

from the viewer should be further along the viewing axis.

5B: Give a Lookfrom/Lookat/VUP (the positions of 2 points and one vector, all in 3D) that specifies a camera that would draw a red triangle pointing to the right, with its tip at the origin.

	<p>Look From: (-10,1,1) (could be in many places to the “west”) Look At: (1,1,1) Vup: (0,1,0) 2 points for each point = 6 points total</p>
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Question 9 (6 pts)

3 pts each

Describe in words what each of these 4x4 transformation matrices does to objects:

A) **Translate the Object by 3,0,0 (3 along X)**

B) **Nothing (after homogeneous divide).**

Note: to get credit for saying scaling, you must specify homogeneous space

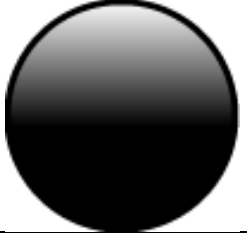
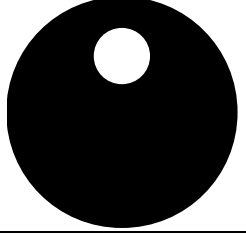
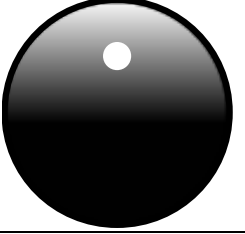
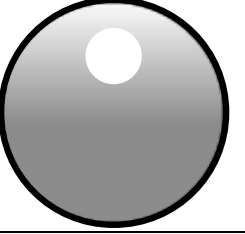
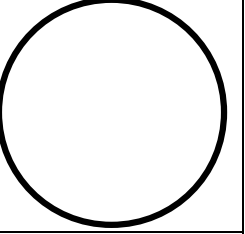
Question 10: (12 points)

Lit from above using the Phong lighting model, a shiny sphere looks (approximately) like:



The light source is straight above the sphere, and the camera is viewing the sphere horizontally.

Sketch how the sphere would look with:

				
A) No Specular Lighting	B) No Diffuse Lighting	C) With the shininess increased to a large value	D) more ambient lighting	E) Lots of Ambient Light (and no diffuse or specular)
No dot	Black except for dot (dot might fall off)	Smaller dot	Overall brighter everywhere	Solid and bright

3 pts each for A,B,C
2 pts each for D,E