CS559 Introduction to Computer Graphics – Spring 2015

Practice Midterm

1. MULTIPLE CHOICE SECTION. Circle or underline the correct answer (or answers). You do not need to provide a justification for your answer(s).

   (1) Which of the following are legitimate reasons for the popularity of *cubic* polynomial curves?
   (Circle or underline ALL correct answers)

   (a) Cubic curves give us the ability to exactly capture the shape of circles or circular segments.

   (b) Cubics are the lowest-degree polynomial that allows us to create curves that are not straight lines.

   (c) It is easier to create a spline curve with C(2) continuity when joining together cubic curves (as opposed to, say, using quadratic curves).

   (2) Which of the following are legitimate reasons for using spline curves (composed of many polynomial curves of small degree, properly connected), as opposed to simply using polynomials of higher degree?
   (Circle or underline ALL correct answers)

   (a) Polynomial curves of very high degree can get uncontrollably wiggly.

   (b) With a single polynomial, even if it has high degree, we can only control what points it goes through, but cannot control its derivative at such points.

   (c) Spline curves can offer *adequate* continuity at modest cost. The extra continuity of high-order polynomials carries added cost, and might not be visually discernible from what a spline curve provides.

   (3) When combining transforms by multiplying the respective 4×4 matrices, the *order* of multiplication (i.e. the order of composition) typically matters. In which of the following cases can we, as an exception, swap the order between two sequentially applied transformations?
   (Circle or underline ALL correct answers)

   (a) A translational and a scaling transformation.

   (b) Two scaling transformations.

   (c) A translation and a perspective projection transformation.

   (d) Two rotation transformations in 3D.

(4) Which of the following visual features is characteristic of using area lights?
(Circle or underline the ONE most correct answer)

(a) Specular highlights.
(b) Soft shadows.
(c) Smooth-shaded objects.

(5) Given that perspective projection is closer to how our human visual system works (as well as cameras, etc), why would we ever consider using orthographic projection?
(Circle or underline the ONE most correct answer)

(a) The cost of rendering images using orthographic projection is substantially lower.
(b) In the absence of perspective projection, all transforms can simply be described as 4×4 matrices.
(c) Orthographic projection keeps lines that are parallel in the world, parallel on the screen. This can be useful for, say, engineering and architectural applications.

(6) Which of the following are true statements about Bezier curves?
(Circle or underline ALL correct answers)

(a) They interpolate all of their control points.
(b) A Bezier curve whose control points are on a straight line, would necessarily be a straight line itself.
(c) The tangent of the curve at the starting location is parallel to the line segment formed by the first two control points.

(7) Which of the following would be a good test for checking that the angle between points $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ equals 90 degrees?
(Circle or underline the ONE most correct answer)

(a) A dot product test : $(\mathbf{p} - \mathbf{q}) \cdot (\mathbf{q} - \mathbf{r}) = 0$
(b) A cross product test : $(\mathbf{p} - \mathbf{q}) \times (\mathbf{q} - \mathbf{r}) = \vec{0}$
(c) A cross product test : $(\mathbf{p} - \mathbf{q}) \times (\mathbf{p} - \mathbf{r}) = \|(\mathbf{p} - \mathbf{q})\| \|(\mathbf{p} - \mathbf{r})\|$ (where $\|\mathbf{x}\|$ is the magnitude of the vector $\mathbf{x}$)

2. SHORT ANSWER SECTION. Answer each of the following questions in no more than 1-2 sentences.

   (a) Write a mathematical expression that can be used to check if 3 points $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are *collinear*.

   (b) Give an intuitive description of what kinds of visual effects one can achieve by using area lights, that are not possible with point lights or directional lights.

   (c) What is our incentive in trying to express projection as a matrix multiplication?

   (d) Name 2 reasons for using homogeneous coordinates to encode locations of points in 3D.

   (e) If a 4×4 transformation matrix is *diagonal* (and its lower-rightmost entry is equal to one), what kind of geometric transformation does it correspond to? Be precise.

3. In 3 dimensions, consider a *cubic* parametric curve $\mathcal{C}(t)$ that satisfies the following constraints:

   - $\mathcal{C}(0) = \mathbf{p}_0$
   - $\mathcal{C}'(0) = 3(\mathbf{p}_1 - \mathbf{p}_0)$
   - $\mathcal{C}''(0) = 9(\mathbf{p}_2 - 2\mathbf{p}_1 + \mathbf{p}_0)$
   - $\mathcal{C}(1) = \mathbf{p}_3$

   where $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are points given as input. Write the parametric curve in the form

   $$\mathcal{C}(t) = b_0(t)\mathbf{p}_0 + b_1(t)\mathbf{p}_1 + b_2(t)\mathbf{p}_2 + b_3(t)\mathbf{p}_3$$

   by giving concrete expressions for the basis functions $b_0(t), \ldots, b_3(t)$.

4. Consider the helical parametric curve, in 3D, given as

   $$\mathcal{C}(t) = [x(t) \;\; y(t) \;\; z(t)] = [cos(t) \;\; sin(t) \;\; t], \quad t \in [0, 10\pi]$$

   (a) Convert this curve to an equivalent, arc-length parameterized curve $\hat{\mathcal{C}}(s)$

   (b) For every value of the parameter t, write an expression (as a function of t) of the *vector* that is perpendicular to $\mathcal{C}(t)$ and points towards the axis of symmetry of the helix (the z-axis). [Hint: It might be useful to draw a picture of this scenario]. Although a mathematical proof/derivation is preferred, an intuitive derivation is also acceptable if well-explained.

5. One (of many) ways to express that 3 points in space are *collinear*, is to write them as a weighted average of one another. That is, points $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$ are collinear if and only if we can write

   $$\mathbf{p} = (1 - \lambda)\mathbf{q} + \lambda\mathbf{r}$$

   [For example, if $\lambda = 0.5$, $\mathbf{p}$ is located halfway in between $\mathbf{q}$ and $\mathbf{r}$. If $\lambda = 0.25$, then $\mathbf{p}$ is still located on the line segment from $\mathbf{q}$ to $\mathbf{r}$, at a distance from $\mathbf{q}$ equal to $1/4$ of the line segment]

   Give a concrete proof that, after applying a *perspective* transformation on the 3 points, their transformed versions will *still* be collinear (albeit, with a possibly different λ value).