Animating Images with Drawings
Litwinowicz and Williams - SIGGRAPH '94

Some base problem -
small # of control points
deform 2D
Warp whole plane (to modify drawing)

Scattered Data Interpolation
Shepard's Interpolant (simple RBF)
Thin plate Spline: \( \min \frac{\partial^2 F}{\partial^2 v} + \frac{\partial^2 F}{\partial u \partial v} + \frac{\partial^2 F}{\partial v^2} \)
NAIVE approaches (triangulation)
Marching Squares
Triangulations

**MINIMIZE DISTORTION** -
Change in triangle shapes (each rigid)
non-linear (preserves lengths)

given - how control points move
find - where the rest of the points go

uses only rest shape (why is this good/bad?)

(1) $v_2$ in coord system of $v_0$, $v_1$

![Diagram]

Note: only works in 2D!
(since $v_0$, $v_1$, $v_2$ must be a line)

error per vertex
(sum of 3 errors per triangle)

Note: invariant to rotation, translation, uniform scale

As similar as possible!

Error is quadratic - minimize w/ linear system
What is SORNIE '04 (Laplacian) Method

Laplacian Surface Editing

Define vector between vertex and average of neighbors

\[ V_i = L_i + \frac{\sum_{j \in N} V_j}{N} \]

Laplacian Coordinate

Find \( V' \) for some \( V_i = V_i' \)

\[ \Delta_i = L(V_i') - L_i \]

\[ \text{minimize this } \sum ||\Delta_i||^2 \]

Problem: not coordinate invariant
Idea: transform \( V_i \) \( \leftrightarrow \) find the transform that minimizes

\[ \Delta_i = L(v_i') - T_i L_i \]

Note: 2 unknowns
but still linear
2) **Minimizing Change in Scale**

- **For Each Triangle**

  CAN FIND CLOSEST SIMILAR TRIANGLE

  \[ V_2 = F(V_0, V_1) \]  \( \text{\text{using triangle coordinates}} \)

  can then rescale (divide by change in length)

  but the triangles have become disconnected

2b) **minimize edge differences**

- same edges as collection of triangles from 2
- \( \text{\text{connected mesh}} \)

**Bonus Pieces**

- Depth
- Local Stiffness
- Performance Animation (real-time dragging)
- Curves (dynamically grow influence region)
ARAP INTERPOLATION (Alexa '00)

\[ \begin{align*}
& \text{factor into 2 parts:} \\
& \text{rigid, non-rigid} < \text{factor matrix into rot, trans, scale, shear}
\end{align*} \]

Each triangle is separate -
then least square tie them together